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OFFICE NOTE 68

More on Detection and Correction of Errors in Height and Temperature Analyses

Frederick G. Shuman
National Meteorological Center

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This supplement to Office Note 67 is for the purpose of stating the problem of "error" detection more precisely, and to suggest a resulting modification of the detection and correction procedures. We will retain the notation of Office Note 67, and pick up the equation numbers where Office Note 67 left off, for ease of cross-reference.

Consider the whole set of ten analysis levels from 1000 to 100 mb. Where the subscripts refer to levels as in the Table, we write the error, ϵ, [(2) and (3)],

$$\epsilon_k = \frac{z_{k+1} - z_{k-1}}{\Delta_k} - \frac{T_{k+1} + T_{k-1}}{2\gamma} \tag{10}$$

Now, define a weighted sum of squares of ϵ.

$$I = \sum_{m=1}^{8} \Delta_{2m+1} \, \epsilon^2_{2m+1} \tag{11}$$

The 1000 mb level is not included because no temperature analysis is available there.

As in Office Note 67, we will "correct" only a single z or T in each iteration, with the exception suggested at the end of this note. We will choose the level, and between z and T, on the basis of minimizing I as defined by (11). Differentiate (11) by one of heights, 700 to 150 mb, inclusive.

$$\frac{dI}{dz_{2n}} = 2\left(\Delta_{2n+1}\,\epsilon_{2n+1}\frac{d\epsilon_{2n+1}}{dz_{2n}} + \Delta_{2n-1}\,\epsilon_{2n-1}\frac{d\epsilon_{2n-1}}{dz_{2n}}\right)$$

$$= -2\left(\epsilon_{2n+1} - \epsilon_{2n-1}\right)$$

$$= -\frac{2}{\gamma}\left(\delta_1\right)_{2n} \tag{12}$$

Also differentiate (11) by one of the temperatures, 700 to 150 mb, inclusive.

$$\frac{dI}{dT_{2n}} = -\frac{\Delta_{2n+1} + \Delta_{2n-1}}{2\gamma^2}\left(\delta_2\right)_{2n} \tag{13}$$

In (12) and (13), δ_1 and δ_2 are as defined in (6) and (7).

The quantity I is thus minimized by the procedure of Office Note 67 as shown by (8) and (9), but only if the level at which the change is made is chosen correctly. Our purpose is to choose the level on the basis of the greatest reduction of I .

As implied by (12) and (13), a change of z or T at one level from 700 to 150 mb, inclusive, changes ϵ in two layers, the ones immediately above and below the level. Denoting a new quantity after a change by a prime, $(\ )'$, and for convenience using the subscript notation defined in Office Note 67,

$$I - I' = \Delta_2 (\epsilon_2^2 - \epsilon_2'^2) + \Delta_1 (\epsilon_1^2 - \epsilon_1'^2) \qquad (14)$$

If it is z_o which is changed, we find, by substitution from (8) into (2) and (3), and then into (14), that

$$I - I' = \frac{\Delta_1 \Delta_2}{\gamma^2 \Delta_o} \delta_1^2 \qquad (15)$$

If it is T_o, on the other hand, which is changed, then

$$I - I' = \frac{\Delta_o}{4\gamma^2} \delta_2^2 \qquad (16)$$

Because our purpose is to minimize I , we should therefore choose the level and between z and T , <u>not</u> on the basis of the magnitudes of δ_1 and δ_2 , but on the basis of the magnitudes of D_1 and D_2 , defined below.

$$D_1 = \delta_1 \sqrt{\frac{\Delta_1 \Delta_2}{\Delta_o}} = \sqrt{\frac{\Delta_1 \Delta_2}{\Delta_o}} \left(\gamma \frac{\Delta_1 z_2 - \Delta_o z_o + \Delta_2 z_1}{\Delta_1 \Delta_2} - \frac{T_2 - T_1}{2} \right) \qquad (17)$$

$$D_2 = \frac{\delta_2}{2} \sqrt{\Delta_o} = \frac{\sqrt{\Delta_o}}{2} \left(2\gamma \frac{z_2 - z_1}{\Delta_o} - \frac{\Delta_2 T_2 + \Delta_o T_o + \Delta_1 T_1}{\Delta_o} \right) \qquad (18)$$

Having already calculated D_1 or D_2 , the new value, z_o' or T_o', may be calculated by

$$z_o' = z_o + \frac{D_1}{\gamma} \sqrt{\frac{\Delta_1 \Delta_2}{\Delta_o}} \tag{19}$$

$$T_o' = T_o + \frac{2 D_2}{\sqrt{\Delta_o}} \tag{20}$$

With the objective of minimizing I as defined in (11), the possibility of changing the uppermost and/or lowermost height and temperature analyses (at 100 and 850 mb in our case) is open. There is a problem here, however. The height and temperature at $k = 2$ or $k = 18$ affects only one ϵ according to the limits on the summation in (11). Consequently, there is no clear basis for choosing between height and temperature as being in "error."

Development of the procedure for detection would proceed as follows. First take 100 mb, for example. Differentiating I by z_{18},

$$\frac{dI}{dz_{18}} = 2\Delta_{17}\, \epsilon_{17} \frac{d\epsilon_{17}}{dz_{18}} = 2\, \epsilon_{17}$$

also

$$\frac{dI}{dT_{18}} = - \frac{\Delta_{17}\, \epsilon_{17}}{\gamma}$$

Thus z_{18}' and/or T_{18}' would be calculated so as to make ϵ_{17}' be zero.

The resulting change in I would be

$$I - I' = \Delta_{17}\, \epsilon_{17}^2$$

The "D" at $k = 18$, in order to be compared to the D_1 and D_2 defined by (17) and (18) would be

$$D = \sqrt{\Delta_{17}}\ \gamma\ \epsilon_{17} \tag{21a}$$

and

$$z_{18}' = z_{18} - \frac{D\sqrt{\Delta_{17}}}{\gamma} \tag{21b}$$

or

$$T_{18}' = T_{18} + \frac{2D}{\sqrt{\Delta_{17}}} \tag{21c}$$

The corresponding formulas for 850 mb are (where the subscript "2" denotes 850 mb as in the Table, and does not have the meaning used in Office Note 67)

$$D = \sqrt{\Delta_3}\ \gamma\ \epsilon_3 \tag{22a}$$

$$z_2' = z_2 + \frac{D\sqrt{\Delta_3}}{\gamma} \tag{22b}$$

$$T_2' = T_2 + \frac{2D}{\sqrt{\Delta_3}} \tag{22c}$$

The D defined by (21a) or (22a), when compared to D_1 and D_2 defined by (17) and (18) could usefully serve to indicate whether an "error" was confined to 100 or 850 mb, since such an "error" would yield a higher value of (21a) or (22a) than of (18) or (19). As mentioned earlier, however, we have no basis for choosing between z or T when it comes to changing the analyses at 100 or 850 mb.

If D at 100 or 850 mb is larger than D_1 and D_2 at 150 or 700 mb, respectively, no change should be made at 150 or 700 mb. In spite of the lack of rationale, it is suggested that instead, half of the changes indicated by (21b) and (21c) be made if it is D at 100 mb which is the largest, or by (22b) and (22c) if at 850 mb.

TABLE

k	p(mb)	π	Δ	Δ_o	$\sqrt{\frac{\Delta_1 \Delta_2}{\Delta_o}}$	$\frac{\sqrt{\Delta_o}}{2}$
18	100	2.30259				
17			.40547			
16	150	1.89712		.69315	.41022	.41628
15			.28768			
14	200	1.60944		.51082	.35450	.35737
13			.22314			
12	250	1.38630		.40546	.31676	.31838
11			.18232			
10	300	1.20398		.47000	.33405	.34278
9			.28768			
8	400	.91630		.51183	.35415	.35771
7			.22315			
6	500	.69315		.55962	.36628	.37404
5			.33647			
4	700	.35668		.53163	.35055	.36456
3			.19416			
2	850	.16252		.35668	.29743	.29861
1			.16252			
0	1000	0				